

Universal four-boson system: dimer-atom-atom Efimov effect and recombination reactions

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Abstract Recent theoretical developments in the four-boson system with resonant interactions are described. Momentum-space scattering equations for the four-particle transition operators are used. The properties of unstable tetramers with approximate dimer-atom-atom structure are determined. In addition, the three- and four-cluster recombination processes in the four-boson system are studied.

Keywords Efimov effect · four-particle scattering · recombination

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1 Introduction

Few-particle systems with resonant interactions (characterized by a large two-particle scattering length a greatly exceeding the interaction range) exhibit universal behaviour. Their properties are independent of the short-range interaction details. The theoretical study of such systems was initiated by V. Efimov in 1970 who predicted the existence of weakly bound three-particle states (Efimov trimers) with asymptotic discrete scaling symmetry [1]. The interest in the universal few-particle systems raised after the Efimov's prediction got confirmed in the cold-atom physics experiments [2]. In the present work we summarize some recent theoretical developments for the universal system of four identical bosons.

The existence of two tetramers below each Efimov trimer was predicted in Refs. [3, 4] while their energies and widths were determined very precisely in Refs. [5, 6]. The resulting four-boson energy (E) spectrum as function of $1/a$ is schematically shown in Fig. 1 (see refs. [6, 7] for quantitative relations between a and E that, for a better visualisation, are not preserved here). The above-mentioned tetramers are labeled as $T(n, m)$ where n refers to the associated Efimov trimer and $m = 1, 2$.

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In the present work we focus on a different type of tetramers, namely, the ones having an approximate dimer-atom-atom structure and labeled as $T^d(n, m)$. Their existence was predicted in Ref. [8] as a consequence of the three-body Efimov effect in the three-body system made off a dimer and two atoms. In a special regime near the Efimov trimer intersection with the atom-dimer threshold the atom-dimer scattering length becomes arbitrarily large and greatly exceeds the dimer size. Under these circumstances one may expect to mimic some properties of the few-boson systems using a model that considers the dimer as a pointlike particle. Then in the effective three-body system consisting of a dimer and two atoms there are two atom-dimer pairs with infinite two-body scattering length. In such a three-body model of the four-boson system the three-body Efimov effect occurs, however, with a very large discrete scaling factor [8]. The dimer-atom-atom structure of the resulting Efimov-like states is only an approximation, and we present the first successful attempt [9] to describe them rigorously as four-boson states. Since they lie in the continuum, we use scattering equations for the four-particle transition operators. For a given Efimov trimer the universal properties of the lowest associated tetramer $T^d(n, 0)$ are determined and its impact on the collisions in the four-boson system is discussed.

Furthermore, using the same technical framework we study the recombination processes in the four-boson system, i.e., the four-boson recombination into atom plus trimer [7] and dimer-atom-atom recombination into atom plus trimer or dimer plus dimer [10].

2 Four-boson scattering equations

We employ Alt, Grassberger, and Sandhas (AGS) equations [11,12] for the transition operators that provide an exact description of the four-particle scattering process. In the case of four identical particles there are only two distinct two-cluster partitions, one of $3 + 1$ type and one of $2 + 2$ type. We choose those partitions to be $(12,3)4$ and $(12)(34)$ and denote them in the following by $\alpha = 1$ and 2 , respectively. The corresponding transition operators $\mathcal{U}_{\beta\alpha}$ for the system of four identical bosons obey symmetrized AGS equations

$$\mathcal{U}_{11} = P_{34}(G_0 t G_0)^{-1} + P_{34}U_1 G_0 t G_0 \mathcal{U}_{11} + U_2 G_0 t G_0 \mathcal{U}_{21}, \quad (1a)$$

$$\mathcal{U}_{21} = (1 + P_{34})(G_0 t G_0)^{-1} + (1 + P_{34})U_1 G_0 t G_0 \mathcal{U}_{11}, \quad (1b)$$

$$\mathcal{U}_{12} = (G_0 t G_0)^{-1} + P_{34}U_1 G_0 t G_0 \mathcal{U}_{12} + U_2 G_0 t G_0 \mathcal{U}_{22}, \quad (1c)$$

$$\mathcal{U}_{22} = (1 + P_{34})U_1 G_0 t G_0 \mathcal{U}_{12}. \quad (1d)$$

Here $G_0 = (E + i0 - H_0)^{-1}$ is the free Green's function of the four-particle system with energy E and kinetic energy operator H_0 , the two-particle transition matrix $t = v + vG_0 t$ acting within the pair (12) is derived from the corresponding potential v , and the symmetrized operators for the $3+1$ and $2+2$ subsystems are obtained from the integral equations

$$U_\alpha = P_\alpha G_0^{-1} + P_\alpha t G_0 U_\alpha. \quad (2)$$

The employed basis states have to be symmetric under exchange of two particles in subsystem (12) for the $3+1$ partition and in (12) and (34) for the $2+2$ partition.

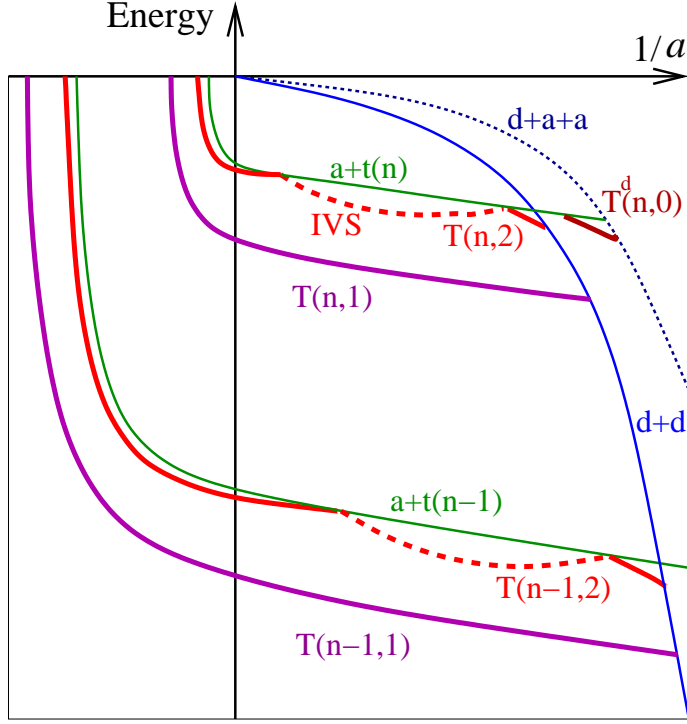


Fig. 1 Schematic representation of the four-boson energy spectrum as a function of the two-boson scattering length. The atom-trimer ($a+t$), dimer-dimer ($d+d$), and dimer-atom-atom ($d+a+a$) thresholds are shown as thin curves. The energies for the standard tetramers (T) and for the tetramers of the dimer-atom-atom structure (T^d) are shown as thick curves. For a better visualization only qualitative but not quantitative relations between them are preserved and only two families of multimers, the $(n-1)$ th and n th, are shown. Positive (negative) a are on the right (left) side from the vertical axis while the four free atom threshold lies on the upper horizontal axis. The dashed parts of the curves for the shallow tetramers $T(n-1,2)$ and $T(n,2)$ correspond to the inelastic virtual states (IVS).

The correct symmetry of the four-boson system is ensured by the operators P_{34} , $P_1 = P_{12} P_{23} + P_{13} P_{23}$, and $P_2 = P_{13} P_{24}$ where P_{ab} is the permutation operator of particles a and b .

All observables for two-cluster reactions are determined by the transition amplitudes

$$\langle \Phi_{\beta,f} | T | \Phi_{\alpha,i} \rangle = S_{\beta\alpha} \langle \phi_{\beta,f} | \mathcal{U}_{\beta\alpha} | \phi_{\alpha,i} \rangle, \quad (3)$$

obtained [13] as on-shell matrix elements of the AGS operators (1); the weight factors $S_{\beta\alpha}$ with values $S_{11} = 3$, $S_{22} = 2$, $S_{21} = \sqrt{3}$, and $S_{12} = 2\sqrt{3}$ arise due to the symmetrization [13]. The matrix elements (3) are calculated between the Faddeev components

$$|\phi_{\alpha,n}\rangle = G_0 t P_{\alpha} |\phi_{\alpha,n}\rangle \quad (4)$$

of the corresponding initial/final atom-trimer or dimer-dimer states $|\Phi_{\alpha,n}\rangle = (1 + P_{\alpha})|\phi_{\alpha,n}\rangle$.

The amplitude for the three-cluster breakup of the initial two-cluster state is given [10] by

$$\langle \Phi_d | T_{d\alpha} | \Phi_{\alpha,n} \rangle = S_{d\alpha} \langle \Phi_d | [(1 + P_{34}) U_1 G_0 t G_0 \mathcal{U}_{1\alpha} + U_2 G_0 t G_0 \mathcal{U}_{2\alpha}] | \phi_{\alpha,n} \rangle \quad (5)$$

with the symmetrization factors $S_{d1} = \sqrt{3}$ and $S_{d2} = 2$ and the dimer-atom-atom three-cluster channel state $|\Phi_d\rangle$. Due to the time-reversal symmetry the three-cluster recombination into a two-cluster state is described by the same amplitude, i.e., $\langle \Phi_{\alpha,n} | T_{\alpha d} | \Phi_d \rangle = \langle \Phi_d | T_{d\alpha} | \Phi_{\alpha,n} \rangle$.

The amplitude for the four-cluster breakup can be obtained [7] as

$$\begin{aligned} \langle \Phi_0 | T_{0\alpha} | \Phi_{\alpha,n} \rangle = & S_{0\alpha} \langle \Phi_0 | (1 + P_1) \{ [1 + P_{34}(1 + P_1)] t G_0 U_1 G_0 t G_0 \mathcal{U}_{1\alpha} \\ & + (1 + P_2) t G_0 U_2 G_0 t G_0 \mathcal{U}_{2\alpha} \} | \phi_{\alpha,n} \rangle \end{aligned} \quad (6)$$

with $S_{01} = \sqrt{6}$ and $S_{02} = 2\sqrt{2}$ and the nonsymmetrized four-boson free-channel state $|\Phi_0\rangle$. Again, due to the time-reversal symmetry the amplitude for the four-boson recombination into a two-cluster state is given by $\langle \Phi_{\alpha,n} | T_{\alpha 0} | \Phi_0 \rangle = \langle \Phi_0 | T_{0\alpha} | \Phi_{\alpha,n} \rangle$.

We solve the AGS equations in the momentum-space partial-wave framework; the technical details can be found in Refs. [7, 13].

3 Tetramers of dimer-atom-atom structure

To obtain universal results we consider reactions involving highly excited Efimov trimers where the finite-range effects become negligible; typically, $n \geq 3$. The $T^d(n, 0)$ tetramer emerges at the n th atom-trimer threshold at the two-boson scattering length $a = 1.608(1) a_n^d$ with a_n^d corresponding to the intersection of atom-trimer and atom-atom-dimer thresholds. As shown in Ref. [9], this tetramer stays very close to the n th atom-trimer threshold and decays through the atom-atom-dimer threshold at $a \approx 0.999999 a_n^d$. The higher tetramers ($n, m \geq 1$) exist only extremely close to $a = a_n^d$ and cannot be resolved in our four-boson calculations; estimations based on the three-body model are given in Ref. [9].

The intersections of the tetramers with the n th atom-trimer threshold manifest themselves most prominently in the collisions of atoms and trimers at vanishing relative kinetic energy leading to the resonant behaviour of the corresponding atom-trimer (n th) scattering length A_n . It is shown in Fig. 2 as a function of the two-boson scattering length a . Here we use $a_n^{dd} = 6.789(1) a_n^d$ to build universal dimensionless ratios; this special value of a corresponds to the intersection of the n th atom-trimer and dimer-dimer thresholds. The A_n resonance at $a_n^{dd}/a \approx 4.22$ is due to the $T^d(n, 0)$ tetramer. The increase of $\text{Re } A_n$ shown in Fig. 2 near $a_n^{dd}/a \approx 6.7$ is not related to the tetramers; it is due to the scaling of the elastic cross section with the spatial size of the trimer that grows with decreasing binding relative to the atom-dimer threshold.

The A_n resonances at $a_n^{dd}/a \approx 0.073$ and $a_n^{dd}/a \approx 0.9984$ correspond to the disappearance and reappearance of the $T(n, 2)$ tetramer as shown in Fig. 1. The explanation for such a spectacular behaviour of the $T(n, 2)$ tetramer may be the following: Increasing $1/a$, i.e., increasing the strength of the two-boson interaction, the binding energies increase for all few-boson states but at different rates. The increase is faster for the dimer, slower for the trimers, and even slower for the tetramers. This is why the n th trimer decays through the atom-dimer threshold at

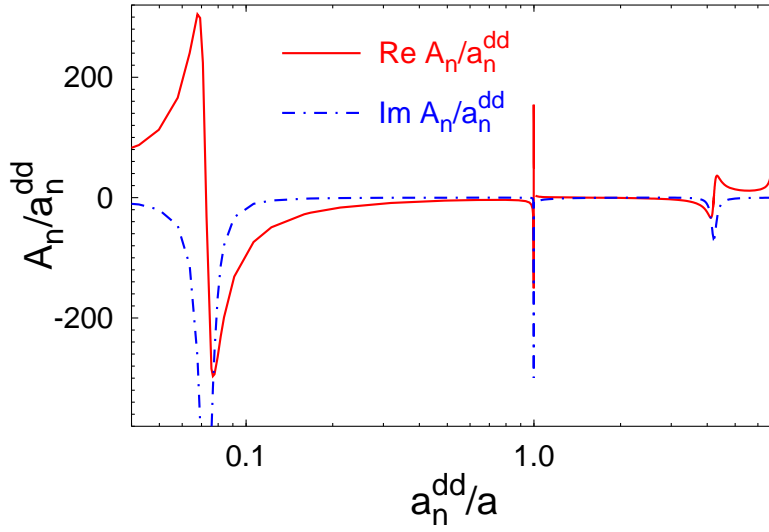


Fig. 2 Atom-trimer scattering length A_n as function of the two-boson scattering length a .

$a = a_n^d$, and the $T(n, 2)$ tetramer, having a structure of an atom weakly attached to the trimer, decays through the atom-trimer threshold at $a_n^{dd}/a \approx 0.073$, becoming an inelastic virtual state (IVS). However, when the n th atom-trimer and dimer-dimer thresholds are close to each other, the interference of the atom-trimer and dimer-dimer configurations in the $T(n, 2)$ tetramer yield additional binding such that the tetramer becomes an unstable bound state until it finally decays through the dimer-dimer threshold [14].

4 Recombination reactions

The number of the four-atom recombination events in a non-degenerated atomic gas per volume and time is $K_4 \rho^4 / 4!$ where ρ is the density of atoms and K_4 is the four-atom recombination rate. Its relation to the breakup/recombination amplitude is given in Ref. [7]. The zero-temperature limit of the four-atom recombination rate K_4^0 is shown in Fig. 3 as a function of the two-boson scattering length. The special value of $a = a_n^0 \approx -3.138 a_n^{dd}$ corresponds to the n th Efimov trimer being at the three free atom threshold. The two peaks of K_4^0 at $a/a_n^0 \approx 0.4254$ and $a/a_n^0 \approx 0.9125$ correspond to the $T(n, 1)$ and $T(n, 2)$ tetramers being at the four free atom threshold, respectively. This is qualitatively consistent with the results of Ref. [4].

The results for the dimer-atom-atom recombination reactions leading to the atom-trimer or dimer-dimer final states are presented in Ref. [10].

5 Summary

We studied the dimer-atom-atom three-body Efimov effect using rigorous four-particle scattering equations for the transition operators that were solved in the

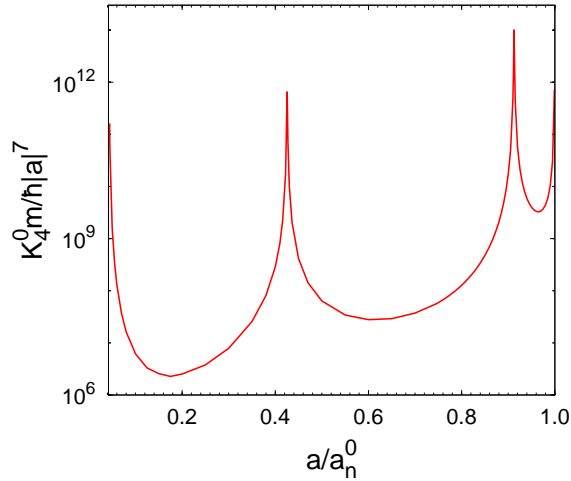


Fig. 3 Ultracold four-boson recombination rate K_4^0 as a universal dimensionless quantity with m being the boson mass.

momentum-space framework. We determined the properties of the deepest resulting tetramers and demonstrated that in ultracold atom-trimer collisions they lead to resonant enhancement of elastic and inelastic reactions. Furthermore, we considered three- and four-cluster recombination reactions in the universal four-boson system and presented results for the ultracold four-boson recombination.

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